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The HIP Algorithm for Track Based Alignment and its Application to the CMS Pixel Detector

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Abstract

Good geometrical alignment is essential to fully benefit from the excellent intrinsic resolution of the CMS silicon tracker. Since the tracker consists of about 20000 independent silicon sensors, of the order 10^5 parameters are needed for the alignment. The determination of these constants with the required precision of about 10 μm is an extremely challenging task. In this paper an effective and computationally practical alignment algorithm is presented. It is suitable for performing fine-calibration of the position and orientation of detector structures consisting of a number of pixel or strip modules as well as the alignment of individual modules. The performance of the algorithm is studied by applying it to the alignment of the CMS Pixel detector.

1 Introduction

Modern silicon tracking detectors such as the CMS tracker [1] are composed of a large number of modules assembled in a hierarchy of support structures. The sensor modules are assembled in ladders or petals. Ladders and petals are in turn assembled in cylindrical or disc-like layers which are then assembled to larger structures to make a complete tracking device. In order not to compromise the tracking performance, the exact positions and orientations of the silicon sensors must be determined with a precision comparable to their very high intrinsic resolution using alignment techniques.

The layout of the CMS tracker is illustrated in Fig. 1. It consists of three cylindrical parts: Inner Barrel (TIB), Outer Barrel (TOB) and Pixel Barrel, as well as of three different disc-like structures: Tracker Endcap (TEC), Tracker Inner Discs (TID) and Pixel Endcaps. The barrel parts are each made of two halves, which are split by the plane $z = 0$ (z-axis in beam direction) for TIB and TOB or by the plane $x = 0$ (x-axis is horizontal) for Pixel Barrel. These halves are further composed of layers (3 Pixel, 4 TIB, 6 TOB) or discs (2 Pixel, 3 TID, 9 TEC) which hold the lowest-level support structures (ladders, rods, wedges and rings [1]) and individual modules. The modules of the two innermost layers/rings of TIB, TOB and TID, as well as TEC rings 1, 2 and 5 provide a coordinate measurement both in $r\phi$ and z.

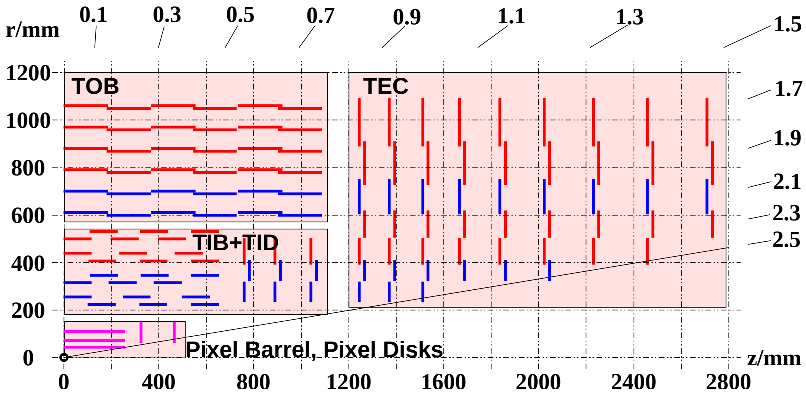

Figure 1: rz-view of one quarter of the CMS silicon tracker.

Sophisticated geometrical calibration is essential in such large detector systems to fully exploit the high intrinsic resolution of the silicon sensors. The positions and orientations of the $\mathcal{O}(20\text{k})$ individual sensors in the CMS tracker have to be calibrated with an accuracy better than the intrinsic resolution of the sensors, which ranges from about 10 μm to 50 μm [1]. The precision to assemble these modules in their position ranges from about 100 μm to a few hundreds of μm [2]. Therefore the position information must be improved by an order of magnitude with calibration procedures.

A laser alignment system is being built for the CMS tracker. It uses infrared laser beams to monitor the positions of selected detector modules of TIB, TOB and TEC [3]. The system can be used to align the TEC discs with respect to each other and to align TIB and TOB as rigid objects with respect to TEC. The Pixel detector as well as the TID are however out of its reach and need to be aligned with tracks. Alignment with particle tracks complements well the optical alignment system, since tracks reach every individual sensor of the detector. Trajectories of high momentum particles are continuous and smooth so their reconstruction residuals, i.e. the differences between the reconstructed trajectory and the recorded hit positions, provide constraints such that the position and orientation of the modules can be optimized using a large sample of events and tracks.

In this paper an effective method by which individual sensors in a detector setup can be aligned to a high precision with respect to each other is presented. This track-based ”Hits and Impact Points” (HIP) method has a long history which started with a silicon strip telescope alignment [4, 5, 6] and it has undergone a steady development since. The formalism described in [6] is extended here to the case of the alignment of composed hierarchical tracker structures, for example rods or layers. The hierarchy of the CMS tracker is illustrated in Fig. 2.

The tracker is divided first to subdetectors and then to parts according to the geometry of their support structures. At lowest level one can find individual modules which are grouped to e.g., Pixel ladders and TOB rods. The algorithm involves iteration over the event sample. During each iteration the track trajectories are kept static which allows to solve the problem at each iteration stage using only small matrices. After each pass over the event sample, the alignment corrections are solved and used in the next iteration over the event sample and the tracks are refitted with the alignment corrections.

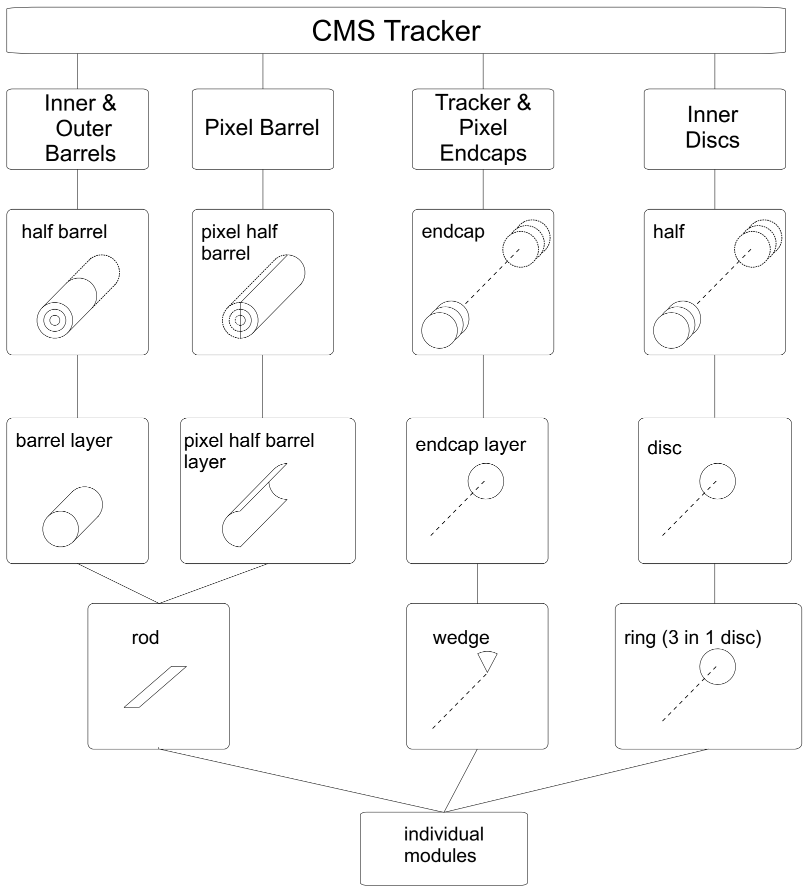

Figure 2: Illustration of the hierarchical structure of the CMS tracker.

The outline of the paper is as follows: In Section 2 basic notations and transformations relevant for the alignment procedures are defined. In Section 3 the detailed formulation of the composite structure alignment is described. This section also includes a short discussion on the relation of the presented algorithm with a single module alignment algorithm. In Section 4 the alignment of the simulated CMS Pixel detector is presented as an example and test for performance.

2 Notations and Transformations

The alignment method as well as the following notations are a generalization of the method presented in [6] in order to take also correlated shifts and rotations of a group of sensors belonging to a composite tracker structure (rod, layer etc.) into account. They reduce to the formalism in [6] in case of no composite rotation or translation.

Conventions and notations for local, composite and global coordinate systems are defined in the following:

$\mathbf{r}$ = coordinates in global system
$\mathbf{g}$ = coordinates in composite system
$\mathbf{q}$ = coordinates in local system
$\mathbf{R}$ = rotation from global to local system
$\mathbf{G}$ = rotation from global to composite system
$\mathbf{r}_0$ = local origin in global coordinates
$\mathbf{g}_0$ = composite origin in global coordinates.

The local coordinates, denoted as $\mathbf{q} = (u, v, w)$, are defined in the sensor (or module) system as follows: the origin is at the center of the detector, the w-axis is normal to the detector, the u-axis is along the precise coordinate and the v-axis along the coarse coordinate. The global coordinates are denoted as $\mathbf{r} = (x, y, z)$. In the case of the CMS detector, the x-axis is horizontal pointing toward the LHC center, the z-axis is parallel to the beam tube axis and pointing to the counterclockwise direction of the LHC ring and the y-axis is upward so as to form a right handed system. The transformations between the coordinate systems are:

$$\mathbf{q} = \mathbf{R}(\mathbf{r} - \mathbf{r}_0) \quad (1)$$
$$\mathbf{g} = \mathbf{G}(\mathbf{r} - \mathbf{g}_0). \quad (2)$$

A composite mis-alignment (unknown translation and rotation) would be corrected by a rotation matrix $\mathbf{\Delta G}$ and a translation vector $\Delta\mathbf{g}$ which would be common to a group of sensors, e.g. belonging to the same support structure. The correction transformation is then:

$$\mathbf{g} \to \mathbf{\Delta G^T g} + \Delta\mathbf{g}. \quad (3)$$

It should be noted that with this correction transformation the corrected $\mathbf{G}$ and $\mathbf{g}_0$ are:

$$\mathbf{G} \to \mathbf{G}_C = \Delta\mathbf{G}^T\mathbf{G} \quad (4)$$
$$\mathbf{g}_0 \to \mathbf{g}_c = \mathbf{g}_0 - \mathbf{G}^T\Delta\mathbf{G}\Delta\mathbf{g}. \quad (5)$$

The correction (3) needs to be propagated to the local system. It follows from the above equations that under the correction (3) the local coordinates $\mathbf{q}$ transform as:

$$\mathbf{q} = \mathbf{R}\mathbf{G}^T\mathbf{\Delta G^T G}\left[\mathbf{r} - \mathbf{g}_0 + \mathbf{G}^T\Delta\mathbf{G}\Delta\mathbf{g} + \mathbf{G}^T\Delta\mathbf{G}\mathbf{G}(\mathbf{g}_0 - \mathbf{r}_0)\right]. \quad (6)$$

Comparing equations (1) and (6) one observes that the corrected rotation $\mathbf{R}_C$ and corrected local origin $\mathbf{r}_c$ read:

$$\mathbf{R} \to \mathbf{R}_C = \mathbf{R}\mathbf{G}^T\mathbf{\Delta G^T G} \quad (7)$$
$$\mathbf{r}_0 \to \mathbf{r}_c = \mathbf{g}_0 - \mathbf{G}^T\Delta\mathbf{G}\Delta\mathbf{g} + \mathbf{G}^T\Delta\mathbf{G}\mathbf{G}(\mathbf{r}_0 - \mathbf{g}_0). \quad (8)$$

The matrix $\mathbf{R}_C$ and the vector $\mathbf{r}_c$ are sensor specific and the corrective rotation $\Delta\mathbf{G}$ and translation $\Delta\mathbf{g}$ are common to the group of sensors to be aligned collectively. It is the task of the alignment procedure to determine the corrections $\Delta\mathbf{G}$ and $\Delta\mathbf{g}$.

3 Composite Alignment Procedure

3.1 Transformation of the Impact Point

The impact point of a particle trajectory at a sensor is a function of the corrections $\Delta\mathbf{G}$ and $\Delta\mathbf{g}$. It is a fairly straightforward calculation to derive this dependence. In the formulation the following notations are used:

$\mathbf{q}_\times$ = the impact point coordinates in sensor system ($\times$ stands for crossing)
$\hat{\mathbf{t}}$ = the trajectory direction at $\mathbf{q}_\times$ in sensor system
$\mathbf{r}_\times$ = the original impact point coordinates in global system
$\hat{\mathbf{s}}$ = the trajectory direction at $\mathbf{r}_\times$ in global system.

The above quantities are given by the track reconstruction in the uncorrected detector. The equation of the trajectory is approximated by a straight line within a (small) range of the alignment correction and it reads:

$$\mathbf{r}(h) = \mathbf{r}_\times + h\,\hat{\mathbf{s}}\,, \quad (9)$$

where h is a variable parameter. A composite rotation and translation causes rotation and translation of the individual sensors which belong to the composite object. When a sensor is rotated and translated, while the trajectory stays static, the impact point $\mathbf{q}_\times$ moves on the sensor plane, so that the third coordinate of $\mathbf{q}_\times$ remains zero. Taking this constraint into account, the value of $h = h_\times(\Delta\mathbf{G}, \Delta\mathbf{g})$ at the intercept is derived:

$$h_\times(\Delta\mathbf{G}, \Delta\mathbf{g}) = -\frac{\mathbf{R}_C(\mathbf{r}_\times - \mathbf{r}_c)\cdot\hat{\mathbf{w}}}{\mathbf{R}_C\hat{\mathbf{s}}\cdot\hat{\mathbf{w}}}, \quad (10)$$

where $\hat{\mathbf{w}} = (0,0,1)$ is a unit vector normal to the sensor and the global vectors $\mathbf{r}_\times$ and $\hat{\mathbf{s}}$ are computed from the local ones as:

$$\mathbf{r}_\times = \mathbf{R}^T\mathbf{q}_\times + \mathbf{r}_0 \quad (11)$$
$$\hat{\mathbf{s}} = \mathbf{R}^T\hat{\mathbf{t}}. \quad (12)$$

The impact point vector in global coordinate system, as a function of the movement $(\Delta\mathbf{G}, \Delta\mathbf{g})$, reads as:

$$\mathbf{r}_\times(\Delta\mathbf{G}, \Delta\mathbf{g}) = \mathbf{r}_\times + h_\times(\Delta\mathbf{G}, \Delta\mathbf{g})\,\hat{\mathbf{s}}, \tag{13}$$

and when transformed to the local sensor coordinates, the formula is:

$$\mathbf{q}_\times(\Delta\mathbf{G}, \Delta\mathbf{g}) = \mathbf{R}_C\left[\mathbf{r}_\times - \mathbf{r}_c + h_\times(\Delta\mathbf{G}, \Delta\mathbf{g})\,\hat{\mathbf{s}}\right]. \tag{14}$$

From the point of view of derivatives calculation a more convenient form of equation (14) is the following:

$$\mathbf{q}_\times(\Delta\mathbf{G}, \Delta\mathbf{g}) = \mathbf{R}_C(\mathbf{r}_\times - \mathbf{r}_c) - [\mathbf{R}_C(\mathbf{r}_\times - \mathbf{r}_c)]_3 \frac{R_C\hat{\mathbf{s}}}{[R_C\hat{\mathbf{s}}]_3}\,, \tag{15}$$

where the subscript indicates the 3^{rd} component of a vector. Furthermore, it is worth noting that

$$\mathbf{R}_C(\mathbf{r}_\times - \mathbf{r}_c) = \mathbf{R}_C(\mathbf{r}_\times - \mathbf{g}_0) + \mathbf{R}\mathbf{G}^T\Delta\mathbf{g} - \mathbf{R}(\mathbf{r}_0 - \mathbf{g}_0). \tag{16}$$

The impact point coordinates enter in the trajectory χ^2 as it will be shown in Section 3.2. Therefore the expression (14) provides a means to optimize $\Delta\mathbf{G}$ and $\Delta\mathbf{g}$ using a large number of tracks and by minimizing the respective χ^2 function. The parameters to be optimized are the three 'tilt' angles and the three translation components:

$$\begin{aligned} \Delta\mathbf{G} &= \Delta\mathbf{G}(\Delta\alpha, \Delta\beta, \Delta\gamma) && (17)\\ \Delta\mathbf{g} &= (\Delta g_1, \Delta g_2, \Delta g_3), && (18) \end{aligned}$$

where $\Delta\alpha, \Delta\beta, \Delta\gamma$ are the three tilt angles and $\Delta g_1, \Delta g_2, \Delta g_3$ are the three correction vector components in the composite coordinate system.

3.2 Solution with the Least Squares Method

The formalism goes very much along similar lines presented in [6]. A measured point in local coordinates is denoted as $\mathbf{q}_m = (u_m, v_m, 0)$. The corresponding trajectory impact point is $\mathbf{q}_\times = (u_\times, v_\times, 0)$. In stereo and pixel detectors two measured coordinates are obtained, u_m and v_m, and in non-stereo strip detectors only one, u_m. In the latter case the coarse coordinate v_m is redundant. The residual $\boldsymbol{\varepsilon}$ is either a 2-vector:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_u \\ \varepsilon_v \end{pmatrix} = \begin{pmatrix} u_\times - u_m \\ v_\times - v_m \end{pmatrix} \tag{19}$$

or a scalar $\varepsilon = \varepsilon_u = u_\times - u_m$. In the following the 2-vector case is formulated. The scalar case follows in a straightforward manner. The χ^2 function to be minimized for a given detector sub-system is:

$$\chi^2 = \sum_i \boldsymbol{\varepsilon}_i^T \mathbf{V}_i^{-1} \boldsymbol{\varepsilon}_i \tag{20}$$

where the sum is taken over the hits i from the tracks traversing the composite unit to be aligned. $\mathbf{V}_i$ is the covariance matrix of the measurements (u_m, v_m) associated to the point i. The alignment corrections, i.e. the three position parameters $(\Delta g_1, \Delta g_2, \Delta g_3)$ and the three orientation parameters $(\Delta\alpha, \Delta\beta, \Delta\gamma)$ are found iteratively by the general or linearized χ^2 minimization procedure because the χ^2 expression is non-linear as a function of the fit parameters.

The set of parameters to be fitted are $\bar{\mathbf{p}} = (\Delta g_1, \Delta g_2, \Delta g_3, \Delta\alpha, \Delta\beta, \Delta\gamma)$. Then, according to the general (linearized) χ^2 solution, the iterative correction to $\bar{\mathbf{p}}$ has the following expression:

$$\delta\bar{\mathbf{p}} = \left[\sum_i \mathbf{J}_i \mathbf{V}_i^{-1} \mathbf{J}_i^T\right]^{-1} \left[\sum_i \mathbf{J}_i \mathbf{V}_i^{-1} \boldsymbol{\varepsilon}_i\right] \tag{21}$$

where $\mathbf{J}_i$ is a Jacobian matrix of $\boldsymbol{\varepsilon}_i(\bar{\mathbf{p}})$:

$$\mathbf{J}_i = \nabla_{\bar{\mathbf{p}}}\, \boldsymbol{\varepsilon}_i(\bar{\mathbf{p}}). \tag{22}$$

An adequate starting point for the iteration is a null correction vector $\bar{\mathbf{p}}$=$\mathbf{0}$. The parameter array $\bar{\mathbf{p}}$ is updated as $\bar{\mathbf{p}} \rightarrow \bar{\mathbf{p}} - \delta\bar{\mathbf{p}}$ during the iterative minimization of χ^2.

In general case of two measurements (u_m, v_m), $\mathbf{J}_i$ is a 6×2 matrix. The dimension of the Jacobian is reduced, if a sub-set of the 6 alignment parameters is fitted while others are kept fixed.

The derivative elements of the Jacobian matrix $\mathbf{J}_i$ are in fact obtained from the derivatives of the expression (15). The derivatives with respect to the translation parameters p_1, p_2, p_3 $(\Delta g_1, \Delta g_2, \Delta g_3)$ are simply:

$$\frac{\partial \mathbf{q}_\times}{\partial p_j} = \mathbf{R}\mathbf{G}^T \hat{\mathbf{e}}_j - [\mathbf{R}\mathbf{G}^T \hat{\mathbf{e}}_j]_3 \frac{\mathbf{R}_C \hat{\mathbf{s}}}{[\mathbf{R}_C \hat{\mathbf{s}}]_3}, \ \ j = 1, 2, 3, \tag{23}$$

where $\hat{\mathbf{e}}_j$ are the unit vectors (1,0,0), (0,1,0) and (0,0,1).

The derivatives with respect to the tilt angles p_4, p_5, p_6 $(\Delta\alpha, \Delta\beta, \Delta\gamma)$ are:

$$\frac{\partial \mathbf{q}_\times}{\partial p_j} = \mathbf{D_j}(\mathbf{r}_\times - \mathbf{g}_0) - [\mathbf{D_j}(\mathbf{r}_\times - \mathbf{g}_0)]_3 \frac{\mathbf{R}_C \hat{\mathbf{s}}}{[\mathbf{R}_C \hat{\mathbf{s}}]_3} - [\mathbf{R}_C(\mathbf{r}_\times - \mathbf{r}_c)]_3 \frac{\mathbf{D_j}\hat{\mathbf{s}}[\mathbf{R}_C \hat{\mathbf{s}}]_3 - \mathbf{R}_C \hat{\mathbf{s}}[\mathbf{D_j}\hat{\mathbf{s}}]_3}{[\mathbf{R}_C \hat{\mathbf{s}}]_3 [\mathbf{R}_C \hat{\mathbf{s}}]_3} \tag{24}$$

where $\mathbf{r}_\times$ is the original (uncorrected) impact point and $\mathbf{D_j}$ are the derivatives of $\mathbf{R}_C$:

$$\mathbf{D_j} = \frac{\partial \mathbf{R}_C}{\partial p_j} = \mathbf{R}\mathbf{G}^T \frac{\partial \mathbf{\Delta G^T}}{\partial p_j} \mathbf{G}, \ \ j = 4, 5, 6. \tag{25}$$

Solving $\hat{\mathbf{s}}$ from (13) and inserting to the last term in (24) the derivative formula simplifies to:

$$\frac{\partial \mathbf{q}_\times}{\partial p_j} = \mathbf{D_j}(\mathbf{r}_\times(\bar{\mathbf{p}}) - \mathbf{g}_0) - [\mathbf{D_j}(\mathbf{r}_\times(\bar{\mathbf{p}}) - \mathbf{g}_0)]_3 \frac{\mathbf{R}_C \hat{\mathbf{s}}}{[\mathbf{R}_C \hat{\mathbf{s}}]_3}, \ \ j = 4, 5, 6, \tag{26}$$

where $\mathbf{r}_\times(\bar{\mathbf{p}})$ is given by equation (13) and the dependence on $(\Delta\mathbf{G}, \Delta\mathbf{g})$ is denoted by $(\bar{\mathbf{p}})$. It is easy to verify that the third vector component of the expressions (23) and (26) is identically zero.

3.3 Special Cases

It is interesting to note that in case $\mathbf{G} = \mathbf{R}$, the above formalism reduces to the module by module alignment formalism described in an earlier paper [6]. In this case the 'structure' contains only one module and one has: $\mathbf{g}_0 = \mathbf{r}_0$, $\Delta\mathbf{G} = \Delta\mathbf{R}$, $\mathbf{g} = \mathbf{q}$. The correction formulas reduce to:

$$\mathbf{R}_C = \mathbf{\Delta R^T R} \tag{27}$$

$$\mathbf{r}_c = \mathbf{r}_0 - \mathbf{R}^T \Delta\mathbf{R} \Delta\mathbf{q}. \tag{28}$$

Also the derivative expressions (23) and (26) undergo a simple change in this special case: $\mathbf{R}\mathbf{G}^T = \mathbf{I}$, i.e. the product $\mathbf{R}\mathbf{G}^T$ erases off from the formulas. It can be concluded that much of common code is valid for both single module and composite structure alignment.

Another special case is when the composite coordinate system is the same as the global system ($\mathbf{G} = \mathbf{I}$, $\mathbf{g}_0 = \mathbf{0}$) for example in the case of barrel layers or forward discs. Then the corrected module rotations and origins are:

$$\mathbf{R}_C = \mathbf{R \Delta G^T} \tag{29}$$

$$\mathbf{r}_c = \Delta\mathbf{G}(\mathbf{r}_0 - \Delta\mathbf{g}). \tag{30}$$

The composite origin in this case differs from the global origin only by a small offset.

4 Alignment Studies Using the CMS Pixel Barrel Detector

The alignment algorithm described above has been implemented within the CMS reconstruction software ORCA [7] using a common alignment software framework. This framework provides a generic interface for all track based alignment algorithms within the CMS software, making use of the hierarchical representation of the tracker structure (see Fig. 2) described in [2]. The alignment software framework provides a convenient management and input/output of alignment parameters and correlations and allows the alignment of both individual modules as well as composed structures (e.g. layers, rods, discs) in a straightforward way by providing the necessary derivatives. Tracker test beam setups are also supported [8]. Misalignment of silicon sensors is implemented after detector simulation at the reconstruction level using a dedicated software tool [2] which is able to move and rotate all Tracker parts (individual sensors as well as composed structures).

A sample of fully simulated and reconstructed $Z^0 \to \mu^+\mu^-$ events are used for the studies. It is estimated that for "low" LHC luminosities of $2 \times 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$, around 20K (100K) $Z^0 \to \mu^+\mu^-$ ($W^\pm \to \mu^\pm\nu$) events are recorded by CMS (i.e. are accepted by the High Level Trigger) per day. Since at the start-up of the LHC the luminosity will be significantly lower, well measured tracks from minimum-bias or QCD jet events should also be considered for the alignment. In addition, muons from $J/\psi \to \mu^+\mu^-$ decays could also serve as a useful source of alignment tracks where, as for the $Z^0 \to \mu^+\mu^-$ events, an invariant mass constraint could be exploited in addition.

The CPU requirements of the HIP algorithm are relatively modest, since there is no inversion of large matrices involved. It can also easily be run in a parallel environment by processing a fraction $1/N$ of the event sample on N machines in parallel, before the alignment corrections are calculated using the combined information and the next iteration is started. For the studies presented in this paper, a special data format was used, such that only tracks used for the alignment were contained. In addition, a refit of already reconstructed tracks was made using hits already found by the pattern recognition. This has been implemented in order to further improve processing time, which is particularly important for iterative alignment algorithms as the one presented here, which require processing the same data sample many times. In this configuration, the CPU time needed is essentially given by the time needed to read in the events and to refit the tracks used for alignment. Using 20 CPUs in parallel, one million $Z^0 \to \mu^+\mu^-$ events could be processed in about 45 minutes on Intel Xeon 3.06 GHz nodes.

In the following, the algorithm presented here is applied to the CMS Pixel barrel detector. An idealized configuration is considered, where misalignments are only present for the translational degrees of freedom. Furthermore, the Pixel endcaps and the CMS Strip tracker are not misaligned and considered as a reference system.

4.1 Alignment of the Pixel Barrel Ladders

In the first application of the alignment algorithm for composite tracker structures, the 90 ladders of the CMS Pixel barrel detector are considered for alignment. Only translational degrees of freedom are considered, which leads to a problem with 270 unknown alignment parameters. A random misalignment of ladders (random flat shifts of size $\pm 200\ \mu$m in the local x,y, and z coordinates) is applied. The Pixel endcaps and the Strip tracker are not misaligned. A sample of 50K $Z^0 \to \mu^+\mu^-$ events has been used and the alignment procedure has been iterated 5 times.

The results of the alignment are presented in Fig. 3, where the residuals are shown for the three global coordinates. In the left hand side the residuals are shown as a function of the iteration; in the right hand side the residual distributions are shown for three values of performed iteration cycles. The algorithm converges very rapidly, and after five iterations the RMS of the residuals are about 2 μm in x and y and about 8 μm in z.

4.2 Alignment of Pixel Barrel Modules

A more realistic situation has been studied, which would correspond to the situation for the LHC first physics data taking. This was done on the basis of the *First Data* misalignment scenario from [2, 9]. It is supposed to simulate the expected conditions during the first data taking of CMS, where the Strip tracker geometry is known from survey measurements and the Laser Alignment System [3] only. However, in this scenario it is assumed that the Pixel detector has already been aligned to some extent using tracks. Therefore, for this study the misalignments applied by the First Data scenario were scaled by a factor 10, in order to simulate more realistically the conditions at the LHC startup.

The misalignment is applied such that the modules of the Pixel detector are not only randomly misaligned with respect to the ladders, but in addition there are also correlated misalignments of ladders and layers. It turns out that in this misalignment configuration, the random misalignments of modules on ladders are bigger than the correlated misalignments of ladders and layers. Therefore, it is preferential to align all 720 individual Pixel barrel modules individually.

A sample of 200K $Z^0 \to \mu^+\mu^-$ events has been used in 10 iterations. The result of this study is shown in Fig. 4. The convergence is again very good for all 720 modules, with RMS values of 7 (23) μm for the x and y (z) coordinates.

4.3 Standalone Alignment of Pixel Barrel Modules

The studies presented in sections 4.1 and 4.2 are idealized in the sense that the Pixel alignment is done with respect to a perfectly aligned Strip tracker. However, the initial alignment of the Pixel will most likely have to be carried

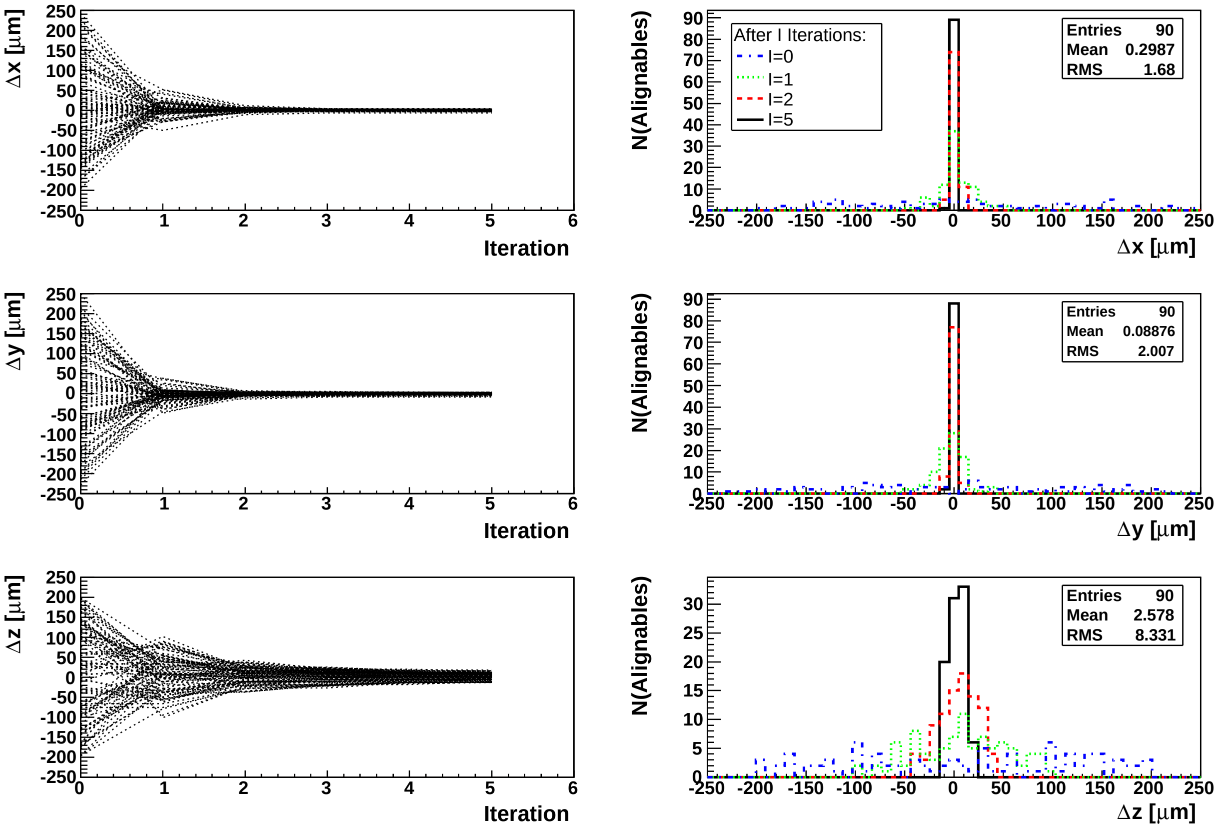

Figure 3: Alignment of the 90 CMS Pixel barrel ladders using the algorithm presented in this note. The residuals in global coordinates are shown as a function of iteration (left) and projected for the initial misalignment (iteration 0) and after 1,2 and 5 iterations (right). The statistical parameters refer to the iteration 5.

out in the presence of a misaligned Strip tracker. Therefore, an attempt has been made to study a standalone Pixel alignment.

In order to avoid a bias originating from the misaligned Strip tracker in the alignment, the procedure adopted here uses a refit of the track parameters taking only Pixel hits into account. In addition, the two muon tracks from the $Z^0 \rightarrow \mu^+\mu^-$ events are fitted to a common vertex (using the standard CMS Kalman Vertex Fitter). The vertex position is then used as an additional constraint in the pixel-only refit of the two tracks. Because of the comparatively small lever arm and only three (two) available measurement layers in the Pixel barrel (endcap) detector, the precision of the track transverse momentum estimate decreases rapidly with increasing p_T ($\sigma(p_T)/p_T \approx 25\%$ for $p_T = 10$ GeV [10]) in the case of a pixel-only track fit. The convergence of the standalone pixel alignment can be significantly improved by using the p_T estimate of the full track fit using the (even poorly aligned) Strip tracker.

Random flat misalignment shifts of ± 300 μm in x,y and z have been applied to the Pixel barrel modules. Since the statistical power when using only Pixel hits in the track fit is much reduced compared with using Strip hits in addition, the convergence of the alignment procedure is slower and more tracks are needed. For this exercise we use 500K events and 19 iterations. The result is shown in Fig. 5. The alignment corrections have been obtained only for 504 out of the in total 720 Pixel barrel modules since tracks are required to have hits in all three Pixel barrel layers.

A good convergence even for this standalone alignment is obtained. The RMS values are around 25 μm for all three coordinates. Although this is not yet sufficiently precise considering the intrinsic resolution of the Pixel modules, the method for the standalone Pixel alignment has been demonstrated to work. The precision of the alignment can be improved by making use of a larger track sample, such as hadrons in minimum bias or jet events.

5 Summary

A detailed formulation of an alignment algorithm for tracking devices is presented and applied to the CMS Pixel detector. The algorithm involves only small matrix inversions and thus requires only little CPU time compared to other alignment algorithms. Standard track reconstruction code of the experiment can be used together with the

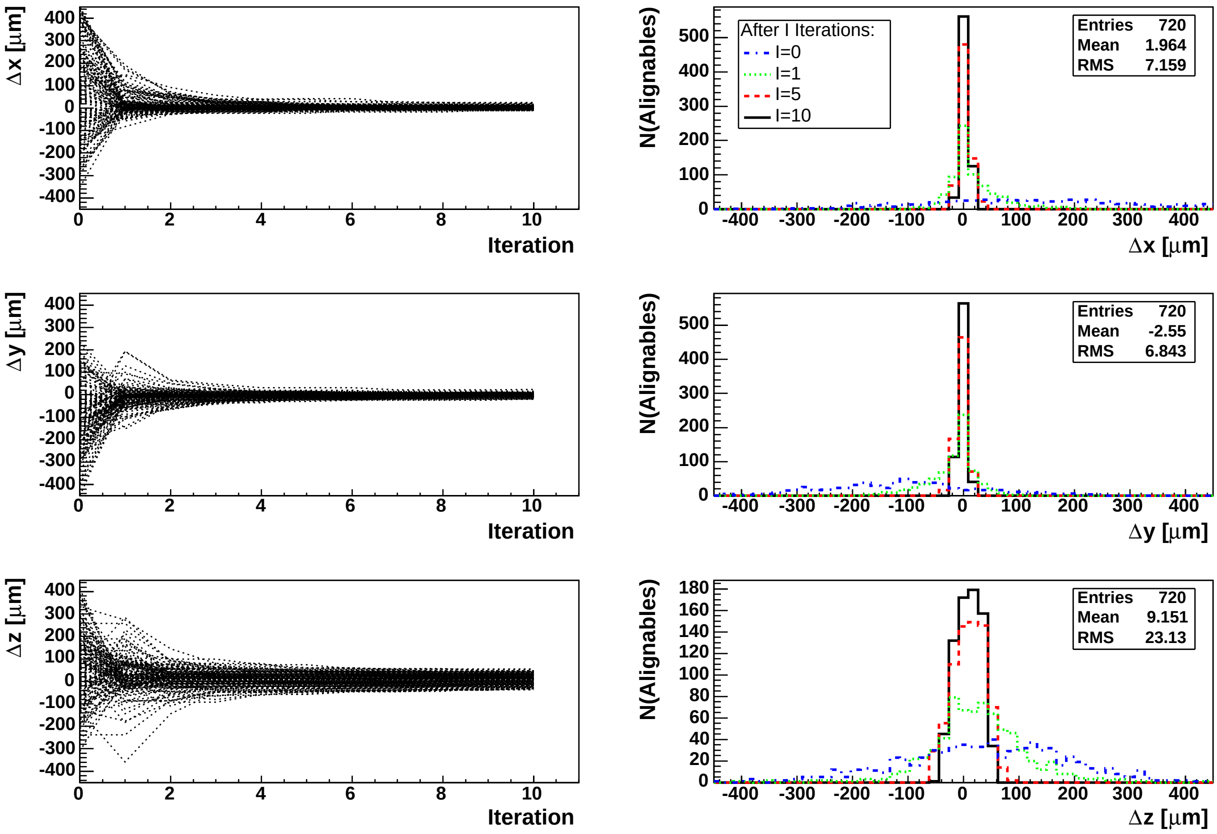

Figure 4: Alignment of the 720 Pixel Barrel modules for the *First Data* misalignment scenario and using the algorithm presented in this note. The residuals in global coordinates are shown as a function of iteration for 144 randomly chosen modules (left) and projected for the initial misalignment (iteration 0) and after 1, 5 and 10 iterations (right). The statistical parameters refer to the iteration 10.

algorithm so that the alignment code itself is very compact. The formulas of the algorithm are such that they apply to both composite structures as well as to individual modules.

The alignment algorithm has been used for alignment studies of the Pixel barrel detector of the CMS tracker. The results show that the algorithm performs well and usually converges within a few iterations. However, it should be noted that more studies using realistic misalignment configurations are needed, among others those which include rotational degrees of freedom, in order to arrive at a realistic alignment strategy for the CMS Pixel as well as for the Strip tracker. In particular, it is likely that the Pixel detector will not be installed in CMS during the first LHC operations. Therefore, a standalone alignment of the Strip detector will have to be studied.

Acknowledgments

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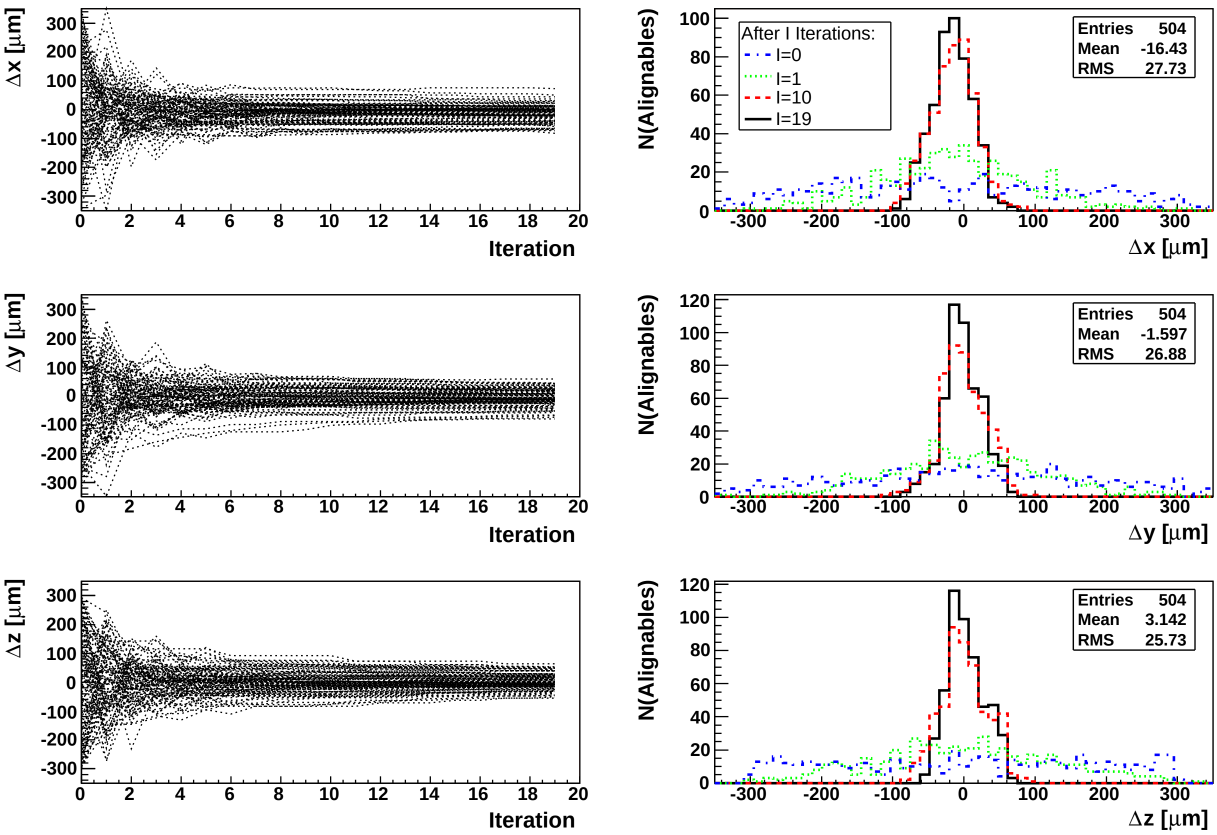

Figure 5: Alignment of 504 out of 720 Pixel Barrel modules. The residuals in global coordinates are shown as a function of iteration for 100 randomly chosen modules (left) and projected for the initial misalignment (iteration 0) and after 1,10 and 19 iterations (right). The statistical parameters refer to the iteration 19.

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